Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Sample Assessment Materials for first teaching September 2018

(Time: 1 hour 30 minutes) | Paper Reference **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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Answer ALL questions. Write your answers in the spaces provided.

1. Use the binomial series to find the expansion of

$$\frac{1}{(2+5x)^3} \qquad |x| < \frac{2}{5}$$

in ascending powers of x, up to and including the term in x^3

Give each coefficient as a fraction in its simplest form.

(6)

① $(2+5x)^{-3}$

$2^{-3}(1+5/2\, x)^{-3}$

$= 1 + (-3)(5/2\, x) + \frac{(-3)(-4)}{2!}(5/2\, x)^2 + \frac{(-3)(-4)(-5)}{3!}(5/2\, x)^3$

$= 1 - 15/2\, x + 75/2\, x^2 - \frac{625}{4} x^3$

$= 2^{-3} = \frac{1}{8}$

$\frac{1}{8}\left(1 - \frac{15}{2}x + \frac{75}{2}x^2 - \frac{625}{4}x^3\right)$

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2. A curve C has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Find an equation of the tangent to C at the point $(3, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

$x^3+2xy-x-y^3-20=0.$

Implicit diff

$3x^2+2y+2x\frac{dy}{dx}-1-3y^2\frac{dy}{dx}=0$

$(2x-3y^2)\frac{dy}{dx}=1-2y-3x^2$

$\frac{dy}{dx}=\frac{1-2y-3x^2}{2x-3y^2}$

(b) $(3,-2)$

$\frac{1-2(-2)-3(3)^2}{2(3)-3(-2)^2}=\frac{11}{3}.$

$(3,-2)$ $m=11/3$

$y=mx+c.$

$-2=\frac{11}{3}(3)+c.$ $c=-13.$

$y=\frac{11}{3}x-13.$

$3y-11x+39=0.$

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3. $$\mathrm{f}(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants A, B and C

(4)

(b) (i) Hence find $\int \mathrm{f}(x)\,\mathrm{d}x$

(ii) Find $\int_1^2 \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in the form $a + \ln b$, where a and b are constants.

(6)

a) $A(3x-1)^2 + B(x)(3x-1) + C(x) = 1.$

let $x=0.$ let $x = 1/3$ let $x=1$

$A=1$ $1/3\,C = 1$ $4A+2B+C$

$\underline{C=3}$ $4(1)+2B+3=1.$

$\underline{\underline{B=-3}}$

(b) i) $\int \frac{1}{x} - \int \frac{3}{3x-1} + \int \frac{3}{(3x-1)^2}$

$= \ln|x| - \frac{\cancel{3}\ln|3x-1|}{\cancel{3}} + 3\int (3x-1)^{-2}.$

$= \ln|x| - \ln|3x-1| - \frac{1}{3x-1} + c.$

(ii) $\left[\ln|x| - \ln|3x-1| - \frac{1}{3x-1}\right]_1^2.$

$\ln|2| - \ln|6-1| - \frac{1}{3(2)-1} = \ln\left(\frac{2}{5}\right) - \frac{1}{5}.$

$\ln(1) - \ln(2) - \frac{1}{2} = \ln\left(\frac{1}{2}\right) - 1/2.$

$\left[\ln(2/5) - \frac{1}{5}\right] - \left[\ln(1/2) - \frac{1}{2}\right] = \ln\left(\frac{4}{5}\right) + \frac{3}{10}$ $a = 3/10$, $b = 4/5.$

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4.

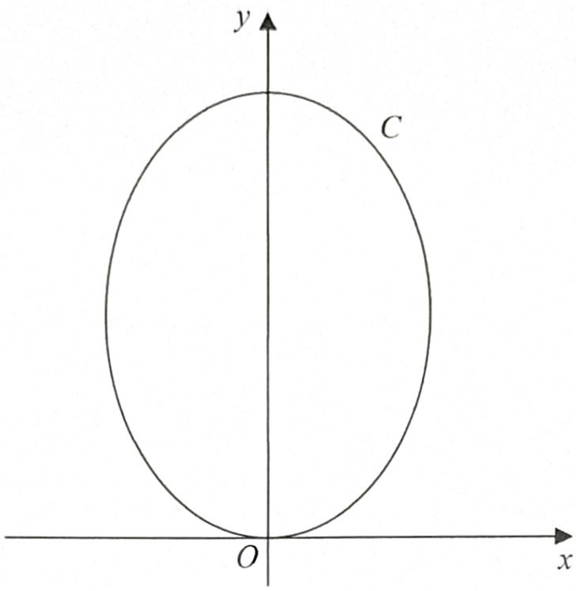

Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = \sqrt{3} \sin 2t \qquad y = 4 \cos^2 t \qquad 0 \leqslant t \leqslant \pi$$

$\cos 2A = 2\cos^2 A - 1$

$\frac{\cos 2A + 1}{2} = \cos^2 A$.

(a) Show that $\frac{dy}{dx} = k\sqrt{3} \tan 2t$, where k is a constant to be found.

(5)

(b) Find an equation of the tangent to C at the point where $t = \frac{\pi}{3}$

Give your answer in the form $y = ax + b$, where a and b are constants.

(4)

(a) $x = \sqrt{3} \sin 2t \quad y = 4\cos^2 t$.

$\frac{dx}{dt} = 2 \times \sqrt{3} \cos 2t = 2\sqrt{3}\cos 2t$

$\frac{dy}{dt}$ of $4\left(\frac{\cos 2t + 1}{2}\right)$

$= 2\cos 2t + 2$.

$\frac{dy}{dt} = -4 \sin 2t$.

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{-4\sin 2t}{2\cos 2t}$

$= -\frac{2}{3}\sqrt{3} \tan 2t \qquad k = -2/3$

(b) $-\frac{2\sqrt{3}}{3} \tan 2t$ at $t = \pi/3$.

$-\frac{2\sqrt{3}}{3} \tan(2 \times \pi/3) = 2$.

$x = \sqrt{3} \sin(2 \times \pi/3) = 3/2$.

$y = 4\cos^2(\pi/3) = 1$.

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Question 4 continued

$m = 2 \quad (3/2, 1)$

$1 = \frac{3}{2}(2) + c \qquad c = -2$

$y = 2x - 2$.

Q4

(Total for Question 4 is 9 marks)

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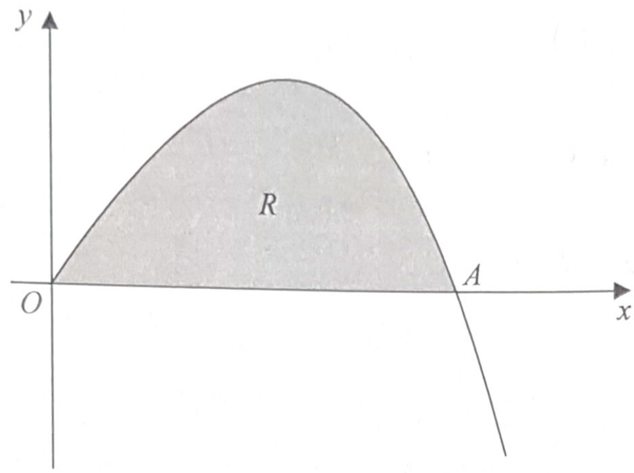

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geqslant 0$

The curve meets the x-axis at the origin O and cuts the x-axis at the point A.

(a) Find, in terms of $\ln 2$, the x coordinate of the point A.

(2)

(b) Find $\int xe^{\frac{1}{2}x}\,dx$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the x-axis and the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geqslant 0$

(c) Find, by integration, the exact value for the area of R.

Give your answer in terms of $\ln 2$

(3)

(a) x axis $(y=0)$

$0 = 4x - xe^{1/2x}$ $\qquad$ $4 - e^{+1/2x} = 0$

$x(4 - e^{+1/2x}) = 0$ $\qquad$ $e^{+1/2x} = 4$

$x = 0$ $\qquad$ $\ln e^{+1/2x} = \ln 4$

$+1/2x = \ln 4$

$x = +2\ln 4$

$x = 2\ln 2^2$

$x = 2\times 2\ln 2 = 4\ln 2$

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Question 5 continued

(b) $\int xe^{1/2x}\,dx$

$u = x \quad du/dx = 1$

$dv/dx = e^{1/2x} \quad v = \frac{e^{1/2x}}{1/2} = 2e^{1/2x}$

$2xe^{1/2x} - \int 2e^{1/2x} = 2xe^{1/2x} - 4e^{1/2x} + c$

(c) $\int_0^{4\ln 2} 4x - xe^{1/2x}$

$\frac{{}^{2}\cancel{4}x^2}{\cancel{2}} - (2xe^{1/2x} - 4e^{1/2x})$

$\left[2x^2 - 2xe^{1/2x} + 4e^{1/2x}\right]_0^{4\ln 2}$

$2(4\ln 2)^2 - 2(4\ln 2)e^{1/2(4\ln 2)} + 4e^{1/2(4\ln 2)} - [2(0)^2 - 2(0)e^{\frac{1}{2}(0)} + 4e^{\frac{1}{2}(0)}]$

$32(\ln 2)^2 - 32\ln 2 + 16 - 4$

$32(\ln 2)^2 - 32(\ln 2) + 12$

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6. Prove by contradiction that, if a, b are positive real numbers, then $a + b \geqslant 2\sqrt{ab}$

(4)

assumption : if a and b are positive real numbers , $a+b \geqslant 2\sqrt{ab}$ is true

$a+b < 2\sqrt{ab}$

squared: $(a+b)^2 < (2\sqrt{ab})^2$

$a^2+2ab+b^2 < 4ab$

$a^2-2ab+b^2 < 0$

$(a-b)^2 < 0$ $\Rightarrow$ the square of any real numbers cannot be smaller than 0

$\therefore$ contradiction

$\therefore$ If a & b are positive numbers , $a+b \geqslant 2\sqrt{ab}$ is true

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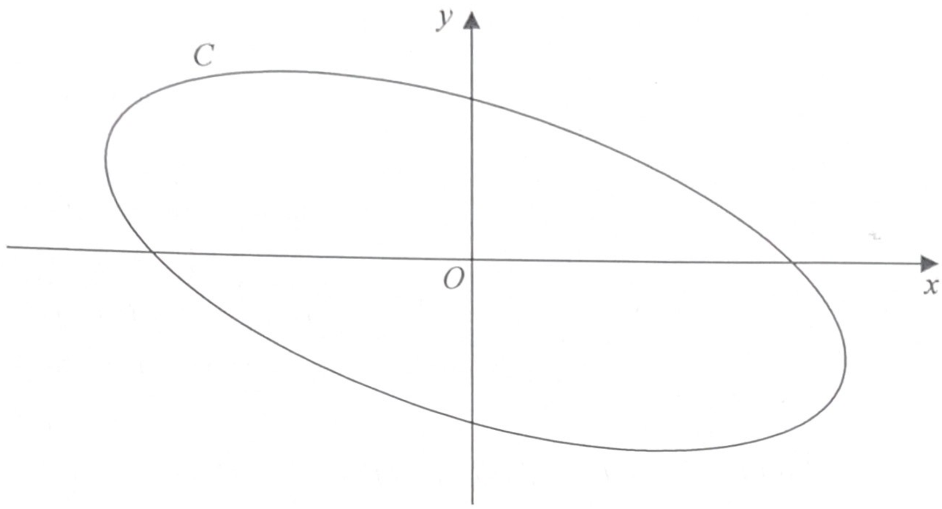

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right) \qquad y = 2\sin t \qquad 0 \leqslant t \leqslant 2\pi$$

(a) Show that

$$x + y = 2\sqrt{3}\cos t$$

(3)

(b) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be found.

(2)

(a) $x = 4\cos(t + \pi/6)$

$4\cos t \cos \pi/6 - 4\sin t \sin \pi/6$

$2\sqrt{3}\cos t - 2\sin t$

$x = 2\sqrt{3}\cos t - 2\sin t$

$x + y = 2\sqrt{3}\cos t$

(b) $\cos t = \dfrac{x+y}{2\sqrt{3}}$ $\quad \sin t = \dfrac{y}{2}$

$\cos^2 t + \sin^2 t = 1$

$\left(\dfrac{x+y}{2\sqrt{3}}\right)^2 + \left(\dfrac{y}{2}\right)^2 = 1$ $\quad \dfrac{(x+y)^2}{12} + \dfrac{y^2}{4} = 1$ $\quad \dfrac{(x+y)^2 + 3y^2}{12} = 1$

$(x+y)^2 + 3y^2 = 12$

$\dfrac{(x+y)^2 + 3y^2}{12} = 1$

$(x+y)^2 + 3y^2 = 12$

$a = 3$

$b = 12$

Leave blank

8. Water is being heated in a kettle. At time t seconds, the temperature of the water is θ °C.

The rate of increase of the temperature of the water at time t is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta) \qquad \theta \leqslant 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$

(a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t}$$

(8)

When the temperature of the water reaches 100 °C, the kettle switches off.

(b) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off.

(3)

8) (a) $\frac{d\theta}{dt} = \lambda(120-\theta)$

$d\theta = \lambda(120-\theta)dt$

$\frac{d\theta}{120-\theta} = \lambda\,dt \qquad \int \frac{1}{120-\theta}\,d\theta = \int \lambda\,dt$

$\frac{\ln|120-\theta|}{-1} = \lambda t + c \qquad -\lambda t = \ln|120-\theta| + c$

$-\lambda(0) = \ln 100 + c$

$-\ln|120-\theta| = \lambda t + c \qquad c = -\ln 100$

$-\lambda t = \ln|120-\theta| - \ln 100$

$-\lambda t = \ln\left|\frac{120-\theta}{100}\right| \qquad e^{-\lambda t} = \frac{120-\theta}{100}$

$100e^{-\lambda t} = 120 - \theta$

$\theta = 120 - 100e^{-\lambda t}$

(b) $\lambda = 0.01$

$\theta = 100°C \qquad \ln\left(\frac{120-100}{100}\right) = -\lambda t \qquad -\lambda t = -1.609437912$

$\lambda t = 1.609437912$

$t = \frac{1.609437912}{0.01} = 160.9s$

$= 161s$

Leave blank

9. With respect to a fixed origin O, the line l_1 is given by the equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

where μ is a scalar parameter.

The point A lies on l_1 where $\mu = 1$

(a) Find the coordinates of A.

(1)

The point P has position vector $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$

The line l_2 passes through the point P and is parallel to the line l_1

(b) Write down a vector equation for the line l_2

(2)

(c) Find the exact value of the distance AP.

Give your answer in the form $k\sqrt{2}$, where k is a constant to be found.

(2)

The acute angle between AP and l_2 is θ

(d) Find the value of $\cos\theta$

(3)

A point E lies on the line l_2

Given that $AP = PE$,

(e) find the area of triangle APE,

(2)

(f) find the coordinates of the two possible positions of E.

(5)

a) $\begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix}$

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Question 9 continued

b) $r = a + \lambda b$

$$r = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

$$r = \begin{pmatrix} 1 + (-5)\lambda \\ 5 + 4\lambda \\ 2 + 3\lambda \end{pmatrix}$$

c) $\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$

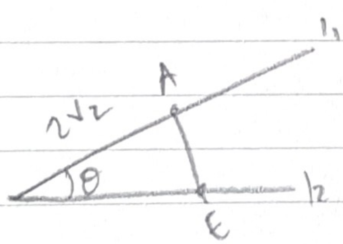
$$\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} = \begin{matrix} -2 \\ 0 \\ 2 \end{matrix} \qquad \sqrt{(-2)^2 + (2)^2}$$

$$= 2\sqrt{2} \qquad k = 2$$

d) $\overrightarrow{AP} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \quad \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}. \qquad \cos\theta = \dfrac{a \cdot b}{|a|\,|b|}$

$$= \frac{16}{2\sqrt{2} \times 5\sqrt{2}} = \cos\theta = \frac{4}{5}$$

(e)



$\cos\theta = \dfrac{4}{5} \qquad \theta = 36.9.$

$$\frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{2} \times \sin 36.9$$

$$= \frac{12}{5}.$$

(f) $|\overrightarrow{PE}| = 2\sqrt{2} \qquad \lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}.$

$$\sqrt{(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2} = 2\sqrt{2}.$$

$$\sqrt{50\lambda^2} = 2\sqrt{2}$$

$$\lambda = \pm \frac{2}{5}$$

$$r = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} \pm \frac{2}{5} \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 33/5 \\ 16/5 \end{pmatrix} \text{ or } \begin{pmatrix} 3 \\ 17/5 \\ 4/5 \end{pmatrix}$$